Warsaw University of Technology

Faculty of Power and Aeronautical Engineering

WARSAW UNIVERSITY OF TECHNOLOGY

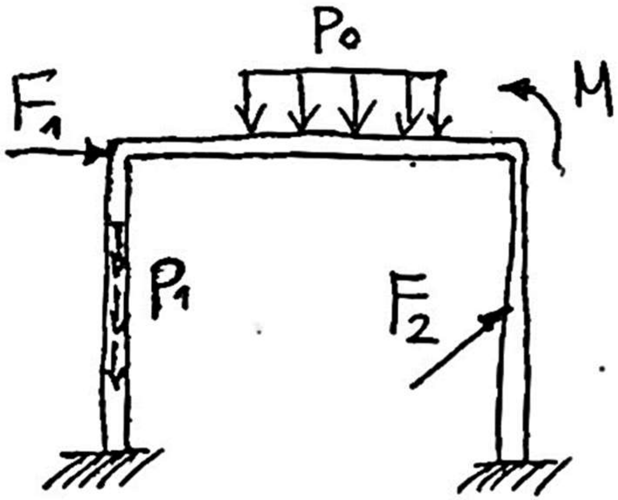
Institute of Aeronautics and Applied Mechanics

Finite element method (FEM)

1D frame finite element

05.2021

PLANE FRAME



Frames – structures made of members rigidly connected. The members can be loaded at any location by concentrated or/and distributed forces, tractions and moments. They carry all possible internal forces: (normal and shear forces, bending moments and torque)

Examples of frames

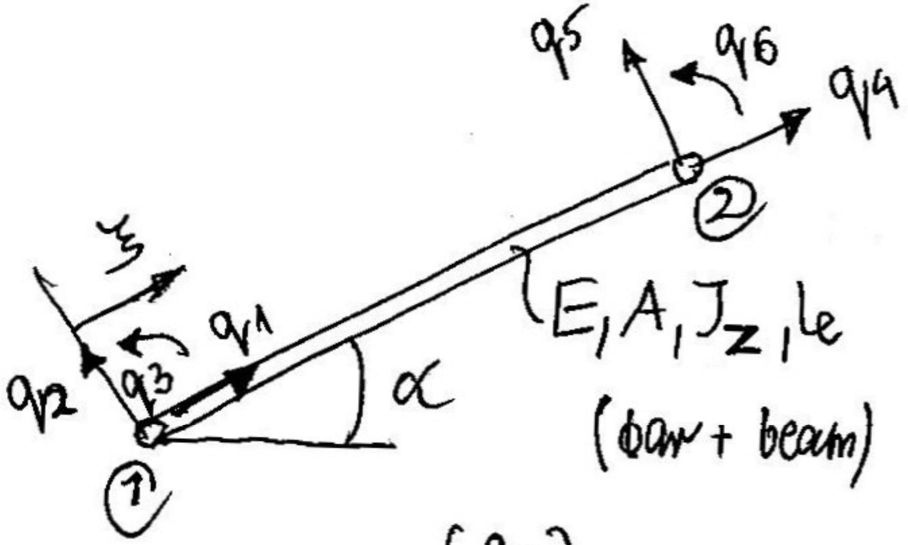
power hang glider

engine mount

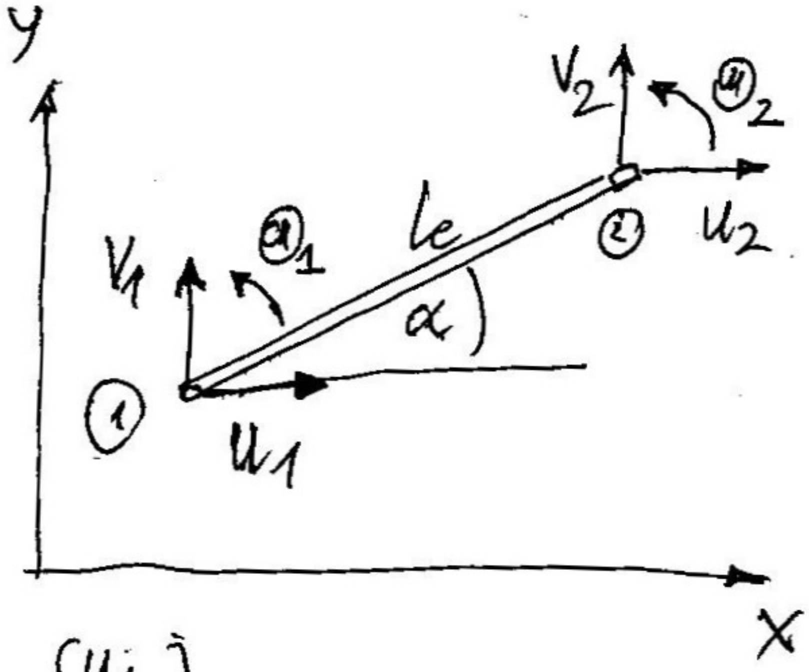
building frame

bike

FRAME ELEMENT



$\Longrightarrow$

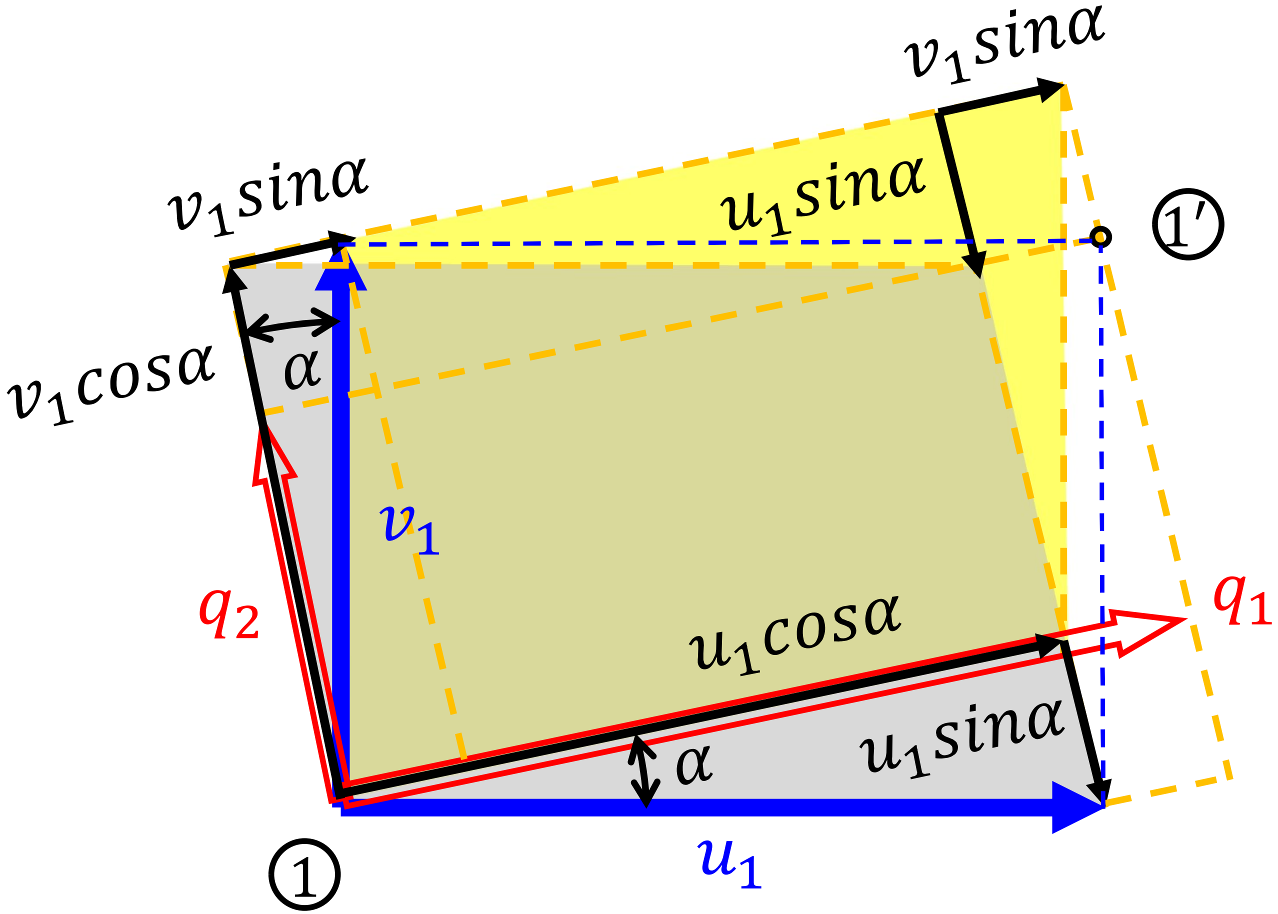


$$\underset{6\times 1}{\{q\}_e} = \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{Bmatrix} \Longrightarrow \underset{6\times 1}{\{q_g\}_e} = \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix}$$

$$\theta_1 = q_3$$

$$\theta_2 = q_6$$

$$c = \cos\alpha$$
$$s = \sin\alpha$$



$$q_1 = u_1 \cdot cos\alpha + v_1 \cdot sin\alpha = c \cdot u_1 + s \cdot v_1$$

$$q_2 = -u_1 \cdot sin\alpha + v_1 \cdot cos\alpha = -s \cdot u_1 + c \cdot v_1$$

$$(c = cos\alpha \quad ; \quad s = sin\alpha)$$

$$q_1 = c \cdot u_1 + s\, v_1 + 0 \cdot \theta_1 + 0 \cdot u_2 + 0 \cdot v_2 + 0 \cdot \theta_2$$
$$q_2 = -s \cdot u_1 + c \cdot v_1 + 0 \cdot \theta_1 + 0 \cdot u_2 + 0 \cdot v_2 + 0 \cdot \theta_2$$
$$q_3 = 0 \cdot u_1 + 0 \cdot v_1 + 1 \cdot \theta_1 + 0 \cdot u_2 + 0 \cdot v_2 + 0 \cdot \theta_2$$

$$q_4 = 0 \cdot u_1 + 0 \cdot v_1 + 0 \cdot \theta_1 + c \cdot u_2 + s \cdot v_2 + 0 \cdot \theta_2$$
$$q_5 = 0 \cdot u_1 + 0 \cdot v_1 + 0 \cdot \theta_1 - s \cdot u_2 + c \cdot v_2 + 0 \cdot \theta_2$$
$$q_6 = 0 \cdot u_1 + 0 \cdot v_1 + 0 \cdot \theta_1 + 0 \cdot u_2 + 0 \cdot v_2 + 1 \cdot \theta_2$$

$$\underset{6\times1}{\{q\}_e} = \underset{6\times6}{[T_f]_e} \cdot \underset{6\times1}{\{q_g\}_e}$$

transformation matrix:

$$\underset{6\times6}{[T_f]_e} = \begin{bmatrix} c & s & 0 & 0 & 0 & 0 \\ -s & c & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c & s & 0 \\ 0 & 0 & 0 & -s & c & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underset{1\times6}{\lfloor q \rfloor_e} = \underset{1\times6}{\lfloor q_g \rfloor_e} \cdot \underset{6\times6}{[T_f]_e^T}$$

$$U_e = \frac{1}{2} \underset{1\times6}{\lfloor q_g \rfloor_e} \cdot \underset{6\times6}{[T_f]_e^T} \cdot \underset{6\times6}{[k]_e} \cdot \underset{6\times6}{[T_f]_e} \cdot \underset{6\times1}{\{q_g\}_e} = \frac{1}{2} \underset{1\times6}{\lfloor q_g \rfloor_e} \cdot \underset{6\times6}{[k_g]_e} \cdot \underset{6\times1}{\{q_g\}_e}$$

where: $$\underset{6\times6}{[k_g]_e} = \underset{6\times6}{[T_f]_e^T} \cdot \underset{6\times6}{[k]_e} \cdot \underset{6\times6}{[T_f]_e}$$

$$[k]_e_{6\times 6} = \begin{bmatrix} \frac{EA}{l_e} & 0 & 0 & -\frac{EA}{l_e} & 0 & 0 \\ 0 & \frac{12EJ_z}{l_e^3} & \frac{6EJ_z}{l_e^2} & 0 & -\frac{12EJ_z}{l_e^3} & \frac{6EJ_z}{l_e^2} \\ 0 & \frac{6EJ_z}{l_e^2} & \frac{4EJ_z}{l_e} & 0 & -\frac{6EJ_z}{l_e^2} & \frac{2EJ_z}{l_e} \\ -\frac{EA}{l_e} & 0 & 0 & \frac{EA}{l_e} & 0 & 0 \\ 0 & -\frac{12EJ_z}{l_e^3} & -\frac{6EJ_z}{l_e^2} & 0 & \frac{12EJ_z}{l_e^3} & -\frac{6EJ_z}{l_e^2} \\ 0 & \frac{6EJ_z}{l_e^2} & \frac{2EJ_z}{l_e} & 0 & -\frac{6EJ_z}{l_e^2} & \frac{4EJ_z}{l_e} \end{bmatrix}$$